Monetary Policy with Parameter Uncertainty in Small-Open Economy*

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Abstract
Recent research in model uncertainty has mainly focused on models of closed economy. In a relevant paper, Giannoni (2002) finds that when the policymaker is uncertain about the numerical value of the structural parameters of the economy, he will optimally respond more aggressively to inflation and output gap fluctuations. That finding contradicts what found in a pioneering paper by Brainard (1967). The aim of this paper is to extend the analysis of uncertainty implications to a new Keynesian small open economy model. In a minimax approach, the central bank derives an optimal policy plan by implementing a Taylor rule. It is shown that if there is uncertainty about the degree of price stickiness and the elasticity of substitution between domestic and foreign goods it is optimal to set interest rates in a less aggressive way. Even if we shut off the foreign channel, by slightly modifying the model, the Brainard principle holds. These results are found for reasonable and experienced degrees of uncertainty in the price stickiness and the elasticity of substitution between domestic and foreign goods.

Keywords: Small open economy; parameter uncertainty; optimal monetary policy; Taylor rule

JEL: E31, E32, F52

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1 Introduction

Central banks typically deal with uncertainty about the true model that explains the economy. This uncertainty reveals through shocks and through any kind of disturbances which makes noisy the information structure available to the policymaker. This paper focuses on the consequences of uncertainty about the slope coefficient of the aggregate supply relationship, hence about the relationship between inflation and the output gap. There are two sources of uncertainty which affects the slope of the aggregate supply relationship: the first one is linked to degree of price stickiness in the economy, while the second one deals with the elasticity of substitution between domestic and foreign goods.

In most of the literature it is assumed that the policymaker knows the true model with certainty. In general, however, policymakers are not aware of the true model of the economy and this is not only a theoretical issue. In fact, uncertainty about the model that governs the economy may lead to disagreement about the effects of monetary policy and so about the appropriate interest rate setting. The uncertainty arises due to structural changes in the economy, the difficulty of observing some important macroeconomic variables or disagreement over competing theoretical models. Since uncertainty can occur in numerous ways, it is important to look for a robust monetary policy which can do a good job even when the policymaker does not know the structure of the economy accurately.

The existing literature on robust monetary policies analyzes mainly closed economy models. However, policymakers do face uncertainty also in open economy, where new channels through which monetary policy can affect the economy add to the ones typical in closed economy. Exchange rate movements can play an important role in the transmission mechanism that links monetary disturbances to output and inflation movements. Moreover, open economies face the possibility of economic disturbances that originate in other countries. These facts constitute a good reason to analyze robust
monetary policies in open economy.

A relevant work which analyzes the conduct of monetary policy under uncertainty is due to Brainard (1967). Brainard evaluates the consequences of uncertainty, expressed in terms of parameter uncertainty, for monetary policy. He finds that if the policymaker is uncertain about the impact a policy instrument has on the economy, it will be optimal to respond more cautiously than would be the case in the absence of uncertainty. Therefore, the policymaker must reduce the magnitude of movements in the interest rate relative to the case of absence of uncertainty. This policy prescription is referred to as the Brainard principle.

More recently, Giannoni (2002) assumes uncertainty about the slope of the Phillips curve and the Euler equation and derives a robust minmax policy that is implemented by a simple instrument rule. By a minmax policy we mean the policy which minimizes the worst possible loss that could occur due to parameter misspecification. Giannoni finds a result which contrasts the Brainard’s one: Policymaker must respond more strongly to inflation than under certainty. Similar results are found by other contributions, eg Onatski and Williams (2003), Söderström (2002) and Onatski and Stock (2002), just to mention a few. Hence robust optimal policy should not obey the Brainard principle anymore.

Therefore, the main aim of this paper is to find out if the Brainard principle holds in open economy, thus if uncertainty about the true structure model calls for a less aggressive policy reaction in open economy. Our results show that Brainard principle holds in small open economy, contrarily to what found recently. To explain our finding, we make reference to the openness of our economic model, and so to the incentive the policymaker has to use the terms of trade strategically, i.e. in a way beneficial to domestic consumers.

However, we conduct our analysis also in the case of closed economy, thus limiting our source of uncertainty in terms of the degree of price stickiness. Even in the latter case, we do find that Brainard principle holds, again in
contrast with the more recent findings in terms of robust monetary policy. If we do not decompose the parameters which hinder our understanding of the Phillips curve slope, but we treat the slope coefficient as an exogenous value, we find that uncertainty implies a more aggressive policy stance. However, in the new Keynesian models the reduced form coefficients depend on several parameters, hence it is not satisfactory not to decompose which are the uncertain parameters which lead to misspecify the structural equations.

The paper is organized as follows. Section 2 describes the key properties of the theoretical model and derives implications for optimal minmax monetary policy under parameter uncertainty. In section 3 we show how to derive the instrument rule - the robust optimal Taylor rule - that implements the optimal minmax equilibrium. Finally section 4 concludes.

2 The Economic Environment

In this section we present the dynamic behavior of a small open economy, like the one developed by Clarida, Galí and Gertler (2001). In particular, the link between the coefficients and the structural parameters is highlighted. Since we are analyzing a problem of uncertain information, we will give details of the information structure in the economy. Then we present the methodology to solve the optimal minmax monetary policy. The methodology follows closely the one developed by Giannoni (2002).

2.1 The Model

The model we are going to use in the rest of the paper has been developed by Clarida, Galí and Gertler (2001). They derive a small-open new Keynesian model which is isomorphic to the closed-economy counterpart. In fact, the key equations are a IS schedule and a Phillips curve:

\[ x_t = E_t x_{t+1} - \frac{1 + \omega}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t) \]  \hspace{1cm} (1)

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_{\pi_t} \]  \hspace{1cm} (2)
where $\pi_t$ denotes domestic inflation rate, $x_t$ stands for the output gap, $i_t$ is the nominal interest rate and $r^n_t$ denotes the natural interest rate, that is, the interest rate that would prevail in equilibrium under flexible prices.

Equation (1) is the IS curve which expresses a negative relationship with the domestic real interest rate and a positive relationship to the expected future output gap. Analogously to the closed-economy counterpart, a rise in the domestic real interest rate reduces aggregate demand and in turn the current output gap via an intertemporal substitution of consumption. In open economy, variation in the domestic interest rate affects also the relative value of the domestic currency: an increase in the domestic real interest rate induces an appreciation of the currency, which reduces aggregate demand through a fall in exports. $\sigma$ indicates the coefficient on relative risk aversion, while $\omega$ depends on some structural parameters. In particular, $\omega \equiv \gamma(\sigma \eta - 1)(2 - \gamma)$, where $\gamma$ is a measure of openness and $\eta$ is the elasticity of substitution between domestic and foreign goods.

Equation (2) is an aggregate supply (AS) curve that relates domestic inflation to the output gap and a cost-push shock $\varepsilon_{\pi_t}$. $\kappa$ depends on some structural parameters according to $\kappa \equiv \frac{(1-\theta)(1-\beta \theta)}{\theta} \left( \phi + \frac{\sigma}{1+\omega} \right)$, where $\phi$ is the inverse of labor-supply elasticity and $\theta$ denotes the degree of price stickiness. In particular, we assume that a fraction $1-\theta$ of firms has the chance to choose their prices optimally. Both shocks are supposed to follow an AR(1) process:

$$r^n_{t+1} = \rho_r r^n_t + \xi_{r_{t+1}} \tag{3}$$

$$\varepsilon_{\pi_{t+1}} = \rho \varepsilon_{\pi_t} + \xi_{\pi_{t+1}} \tag{4}$$

Finally, the terms of trade $s_t$ are proportional to the difference between

\[\beta\] is the discount factor.
domestic and rest of the world’s output: $y_t - y_t^*$:

$$s_t = \frac{\sigma}{1 + \omega} (y_t - y_t^*)$$  \hspace{1cm} (5)$$

This representation of the terms of trade is tantamount to the one proposed by Corsetti and Pesenti (2001) among the others, which present a proportional relationship between the "terms-of-trade gap" and the output gap. Based on that relationship between the terms-of-trade gap and the output gap, a natural policy objective can be expressed as a quadratic loss function in terms of inflation volatility and output gap volatility\(^3\). The policymaker minimizes the following loss function:

$$\min_{\pi t+i + x t+i} \frac{1}{2} \sum_{i=0}^{\infty} \left[ \pi^2_{t+i} + \lambda_x x^2_{t+i} \right]$$ \hspace{1cm} (6)$$

where $\lambda_x > 0$ is the weight attached to the output gap stabilization.

### 2.2 The information structure

Since we want to analyze the impact of parameter uncertainty on the conduct of optimal monetary policy, it is necessary to well define how we chose to model the information structure in the economy. First of all, we decide to model uncertainty about the economy by assuming that there is an asymmetric information structure between the central bank and the private sector. In particular, private sector is assumed to perfectly know the structure of the economy. On the other hand, the policymaker has to formulate optimal policy without knowing the true values of the parameter vector $\mathcal{P} \equiv (\theta, \eta)$. The only information he is provided with is that the coefficients lie in a continuum of parameters like the following:

\(^2\)The rest of the world’s output is modeled as an AR(1) process according to

$$y_{t+1} = \rho_y y_t + \xi_{yt+1}$$

\(^3\)To see the exact conditions under which the loss function can be expressed in terms of inflation volatility and output gap volatility, see Benigno and Benigno (2000) and Clarida, Gali and Gertler (2001).

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The reason why we decide to focus on the uncertainty about the degree of price stickiness and the elasticity of substitution between domestic and foreign goods deals with the difficulties in identifying the correct size of a parameter and the diverse calibrated values proposed in literature. In particular, a wide range of values have been employed in the literature to calibrate the degree of price stickiness $\theta$, which gives evidence of how difficult is to identify this key parameter. Moreover, in the literature it is usually used a value of 1 or 1.5 for the elasticity of substitution between domestic and foreign goods $\eta$, while empirical estimates like the ones in Lai and Treffer (2002) suggest us that a more realistic value for that parameter should be around 6.

Uncertainty about $\eta$ and $\theta$ in turn affect the policymaker’s perception of the true structure of the economy, through the parameters $\kappa$ and $\omega$. $\kappa$ is the parameter which identifies the slope of the Phillips curve, i.e. how expectations about future output gap levels affect the inflation level today. $\omega$ affects the transmission mechanism of monetary policy, hence an incorrect value for it makes the policy maker underestimate or overestimate the effect of a policy choice.

2.3 Optimal monetary policy in a minmax approach

In this section we discuss how we can find the optimal policy which takes into account uncertainty about the true structure of the economy. The solution method follows Giannoni (2002). The policy maker has to choose a policy $f$ before the vector parameter $p \in \mathcal{P}$ is known. The policy $f$ belongs to the set of non inertial rules; since we are in a linear-quadratic setup, we can restrict our attention to linear policy rules. Central bank chooses $f$ to minimize (6), but he does not know the parameter vector $p$. He only knows the lower and upper bound of the interval the parameter vector lies in, hence the only available possibility is minimize the worst possible loss that can occur.
Therefore, the optimal robust policy, denoted by $f^*(p)$, can be defined as

$$f^*(p) = \min_{f \in F} \left\{ \max_{p \in P} E \left[ L_t \left( f(p), p \right) \right] \right\}$$  \hspace{1cm} (7)

where $F$ stands for the entire set of feasible rules characterized by non-inertia and linearity.

The central bank is assumed to commit credibly to implement the non-inertial plan via a simple rule with the properties defined before, namely a Taylor rule like the following:

$$i_t = \phi_\pi \pi_t + \phi_x x_t$$  \hspace{1cm} (8)

Hence, given the minmax optimal plan $f^*$ and the true parameter values, unknown before the policy rule (8) is implemented, equilibrium inflation and output gap are function of the instrument rule previously defined. The minmax equilibrium allocation will be the pair $\{\pi_t \left( f^* (p^*), p^* \right), x_t \left( f^* (p^*), p^* \right)\}$, where $f^* (p^*) \in F$ is the robust policy plan and $p^*$ is the parameter configuration which maximizes the welfare loss. Following Giannoni (2002), the solution strategy involves three steps. First, we find the policy plan for a given parameter vector $f(p)$. Second, nature selects the parameter vector $p \in P$ such that the welfare loss is maximized. Finally, we choose an optimal instrument rule that implements the minmax equilibrium values for inflation, output gap and interest rate.

### 2.4 Finding the optimal policy

The central bank has to derive the optimal policy plan $f(p)$ and the corresponding optimal instrument rule which implements this plan. As we said before, the first step of our solution strategy implies that we find the optimal policy given a general parameter configuration.

Since we are looking for a non-inertial optimal policy plan, we know that the optimal trade-off between $x_t$ and $\pi_t$ can be expressed with the following generic expression:

$$x_t = -f(p)\pi_t$$  \hspace{1cm} (9)
The optimal trade-off (9) can thus be formulated in terms of variances such that

\[ \text{var}(x_t) = f^2(p) \text{var} (\pi_t) \] (10)

Both (9) and (10) are in terms of the model parameters. Combining the last two equations with the new keynesian Phillips curve, we are able to express \( \pi_t \) and \( x_t \) in terms of the supply shock \( \varepsilon_{\pi_t} \):

\[ \pi_t = \frac{1}{1 + \kappa f - \beta \rho_n} \varepsilon_{\pi_t} \] (11)

\[ x_t = -\frac{f}{1 + \kappa f - \beta \rho_n} \varepsilon_{\pi_t} \] (12)

Bearing in mind that the equilibrium we are characterizing is a discretionary one, we can express the loss function in terms of the policy plan \( f(p) \)

\[ \mathcal{L}_t = \frac{1 + f^2(p)}{(1 + f(p)\kappa - \beta \rho_n)^2} \] (13)

In the first step, we find \( f \) by minimizing (13) with respect to \( f \):

\[ f^*(p) = f^*(\theta, \eta) = \frac{(1 - \theta)(1 - \beta \theta)}{\theta (1 - \beta \rho_n)^2} \left( \phi + \frac{\sigma}{1 + \gamma(\sigma \eta - 1)(2 - \gamma)} \right) \] (14)

In the case of perfect information about the true structure of the economy, (14) will provide us with the optimal plan that minimizes the welfare loss. In our case, however, the policymaker has no clue about the true value of \( \eta \) and \( \theta \), hence he can only minimize the loss function in the worst scenario. Hence, after deriving the shape of the optimal plan, in the second step we will find the worst-case parameter configuration, i.e. the values of \( \eta \) and \( \theta \) a fictitious evil agent will choose in order to maximize (6) or (13).

Therefore the policy-maker can conjecture the worst parameter constellation, which solves the following problem

\[ \max_{\eta, \theta} \left\{ \frac{1 + \lambda_x f^2}{(1 - \beta \rho_n + f \kappa)^2} \right\} \] (15)

The first order conditions reveal that

\[ \frac{\partial \mathcal{L}_t}{\partial \eta} > 0 \quad \text{and} \quad \frac{\partial \mathcal{L}_t}{\partial \theta} > 0 \] (16)
so that the worst parameter configuration corresponds to the pair \((\eta_h, \theta_h)\),
that is to the upper bounds of the parameter sets \([\eta_l, \eta_h]\) and \([\theta_l, \theta_h]\).

We have just found that the optimal policy is robust if it takes into
account the previously derived worst case in terms of uncertain parameters.
We can reexpress (14) as a function of the worst parameter configuration
\(\eta_h, \theta_h\):

\[
f^*(p^*) = \frac{(1 - \theta_h)(1 - \beta \theta_h)}{\theta_h(1 - \beta \rho_\pi)} \left( \phi + \frac{\sigma}{1 + \gamma (\sigma \eta_h - 1)(2 - \gamma)} \right)
\]  

(17)

Notice that in (17) we express the optimal policy as a function of \(p^*\)
instead of \(p\) (as in (14)) because we are now considering an optimal robust policy.

2.5 Policy and uncertainty

In the previous section we have derived analytically the main properties of
the equilibrium. To give an idea of how uncertainty affects the equilibrium
response of inflation, output gap and interest rate we calibrate the model
according to the usual values used in literature and some empirical results
which are at odds with them. Starting with the parameters which are cer-
tain, the constant relative risk aversion parameter is set to \(\sigma = 1\), while
the inverse of the labor supply elasticity \(\phi\) is equal to 1.5. The degree of
openness \(\gamma\) is assumed to be 0.6. The discount factor \(\beta\) and the weight of
output gap \(\lambda_x\) follow the common size in literature, namely 0.99 and 0.25
respectively.

Now we consider the two uncertain parameters, \(\eta\) and \(\theta\). The baseline
value for \(\eta\) is 1.5, as commonly assumed in literature. Empirical estimates
of the elasticity of substitution between domestic and foreign goods often
suggest a value of \(\eta\) much bigger than the one previously quoted. For exam-
ple, Lai and Treftler (2002) estimate this parameter equal to 6. Therefore,
we decide to choose 6 as the upper bound of the elasticity of substitution
between domestic and foreign goods\(^4\). The baseline value for price stick-

\(^4\)The lower bound \(\eta_l\) is set equal to one.
iness, \( \theta \), is set to 0.6; however a wide range of values has been employed in literature to capture the size of price stickiness. The uncertainty of \( \theta \) is thus expressed in the interval \([\theta_l = 0.1, \theta_h = 0.9]\). Table 1 summarizes all parameters, including the shocks.

Table 1: Structural parameters of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>CRRA</td>
<td>1</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Labor supply elasticity</td>
<td>1.5</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Openness</td>
<td>0.6</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>( \rho_x )</td>
<td>AR cost-push shock</td>
<td>0.5</td>
</tr>
<tr>
<td>( \sigma_{\varepsilon} )</td>
<td>Std of ( \varepsilon )</td>
<td>1</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>AR natural interest rate</td>
<td>0.4</td>
</tr>
<tr>
<td>( \sigma_r )</td>
<td>Std of ( r^n )</td>
<td>0.01</td>
</tr>
<tr>
<td>( \eta )</td>
<td>elasticity of substitution</td>
<td>1.5</td>
</tr>
<tr>
<td>( \eta_h )</td>
<td>upper bound of ([\eta_l, \eta_h])</td>
<td>6</td>
</tr>
<tr>
<td>( \theta )</td>
<td>price stickiness</td>
<td>0.6</td>
</tr>
<tr>
<td>( \theta_h )</td>
<td>upper bound of ([\theta_l, \theta_h])</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Figure 1 and figure 2 show us what previously found analytically, namely that the welfare loss is increasing both in \( \eta \) and in \( \theta \).
3 Optimal robust Taylor rule

The equilibrium responses of output gap, inflation and interest rate are given by the following relationships:

\[ x_t = -\frac{f^*}{1 + f^* \kappa_h - \beta \rho \pi} \pi_{t} \]  \hspace{1cm} (18)

\[ \pi_t = \frac{1}{1 + f^* \kappa_h - \beta \rho \pi} \pi_{t} \]  \hspace{1cm} (19)

\[ i_t = r^n_t + \left( \rho \pi + \frac{\sigma f^* (1 - \rho \pi)}{1 + \omega_h} \right) \pi_{t} \]  \hspace{1cm} (20)

Not surprisingly, the three above variables depend on the structural shocks. Following the methodology sketched out by Giannoni (2002), we can implement the optimal robust equilibrium allocation with a standard Taylor rule like (8). In order to identify the value of the coefficients \( \phi_x \) and \( \phi \pi \), we plug the equilibrium responses of inflation and output gap into the expression of the Taylor rule and get an expression for the optimal Taylor rule coefficient on inflation \( \phi \pi \) as a function of \( \phi_x \) and the distorted parameters:

\[ \phi \pi = \rho \pi + \kappa \phi_x + \frac{\sigma \kappa (1 - \rho \pi)}{(1 - \beta \rho \pi) (1 + \omega^*)} \]  \hspace{1cm} (21)

In particular, any combination consistent with (21) implements the optimal robust equilibrium described above. Since we are implementing a Taylor rule, i.e. a rule which does not respond to all the states and shocks of the economy, we should check that the uniqueness of equilibrium holds. In particular, appendix A shows that the condition for a unique equilibrium to arise is the following\(^5\):

\[ \phi \pi > 1 - \frac{\beta}{\kappa} \left( \frac{1}{\beta} - 1 \right) \phi_x \]  \hspace{1cm} (22)

Now we analyze one of the main issues of the paper: How does the uncertainty about the true economic structure affect the interest rate response

\(^5\)A graphical illustration of the determinacy properties can be found below.
to inflation, $\phi_\pi$? The reaction of $\phi_\pi$ to $\eta$ and $\theta$ shows that

$$\frac{\partial \phi_\pi}{\partial \eta} < 0 \quad \frac{\partial \phi_\pi}{\theta} < 0$$

Equation (23) represents a key result of the paper: When uncertainty about the price stickiness and elasticity of substitution between domestic and foreign goods increases, a central bank will optimally respond less aggressively to inflation. Therefore in this setup the ”Brainard principle” holds, contrarily to what found by Giannoni (2002). In the rest of the paper, we are going to determine which are the reasons why we have a different result than the Giannoni’s one.

Giannoni finds that optimal robust Taylor rules prescribe a stronger response of the interest rate to fluctuations in inflation and the output gap than in the absence of uncertainty. The setup where he proves this result is a closed-economy one, while we developed our analysis in open economy. The foreign channel is therefore the first natural candidate element which can justify such a different result.

Even if we are considering a model of open economy which is isomorphic to the closed economy counterpart, moving from a closed economy setup to an open economy framework is not an innocuous step. In fact, in closed economy models only cost push shocks impose a policy trade-off, while demand shocks can be completely accommodated without any cost in terms of output gap and inflation. In the open economy models, on the other hand, a policy that moves the short-term nominal interest rate to shield the economy from a demand shock affect also the real exchange rate and in turn the rate of inflation. Therefore the policy-maker cannot achieve inflation and output gap objectives simultaneously\(^6\).

Another fundamental trade-off which arises in open economy is the one which links the output gap to the terms of trade. In a model like the

\(^6\)The difference between closed-economy and open-economy models in terms of policy trade-offs can be seen also considering optimal reaction to other kind shocks, like shocks in the financial markets affecting interest parity condition.
one presented in section 2, there exists the following positive relationship between the output gap and the terms of trade

\[ s_t = \frac{\sigma}{1 + \omega} x_t + \tilde{s}_t \quad \tilde{s}_t \equiv \frac{\sigma}{1 + \omega} (\bar{y}_t - y_t^*) \]  

(24)

where \( s_t \) stands for the terms of trade, \( \bar{y}_t \) is the domestic potential output and \( y_t^* \) represents the rest of the world’s output, which is exogenous in a small-open economy framework.

Equation (24) represents the typical trade-off which is characteristic in open economy between the output gap and the terms of trade. In open economy, in fact, the policymaker has the incentive to use the terms of trade to transfer the burden of production to the foreign counterpart\(^7\). The presence of the output gap in the loss function depends deeply from the direct proportionality between the output gap and the terms of trade. Quantitative analysis below shows that when uncertainty increases, inflation volatility rises but this is more than offset by the reduction in the volatility of the output gap and the terms of trade if the policy maker decides to follow a robust policy.

A more attentive analysis, however, shows that it is not only being in open economy why in our model the "Brainard principle" holds, differently than what found by Giannoni (2002). To that extent, it is straightforward to note that the closed-economy counterpart can be seen as a special case of the model (1)-(2), i.e. when we pick a value of the degree of openness \( \gamma \) equal to zero or a value of \( \eta \) equal to one. In the latter case, uncertainty of the elasticity of substitution between domestic and foreign goods \( \eta \) is irrelevant in our analysis and what central bank cares about in implementing an optimal minmax policy is the uncertainty about the degree of price stickiness \( \theta \). Therefore, two previous results still hold in this case, namely welfare loss is increasing in \( \theta \) and the interest rate response to fluctuations in inflation and the output gap is decreasing in \( \theta \). Therefore, once again we should find

\(^7\)See Benigno and Benigno (2001) and Corsetti and Pesenti (2001), among others.
a rationale for the inverse relationship between uncertainty and degree of policy aggressiveness.

The reason why even in closed economy the "Brainard principle" holds lies in the ability of decomposing the sources of uncertainty. In particular, if we repeat the previous exercise for a given $\kappa$, we find a result which is in contrast to what previously shown. If we write the expression of the optimal Taylor rule response coefficient as a function $\phi_x$ and distorted parameter $\kappa$ in closed economy, we have

$$\phi_\pi = \rho_\pi + \frac{\kappa_\pi^*(\phi_x + \sigma(1-\rho_\pi))}{1-\beta\rho_\pi}$$

$$\kappa_\pi^* \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}(\phi + \sigma)$$

Evaluating (25) in terms of $\kappa_\pi$, we find that the optimal Taylor rule response coefficient is increasing in $\kappa_\pi$, hence uncertainty calls for a more aggressive policy reaction. Notice that the latter finding can be easily extended to the open economy framework: if we do not decompose the uncertainty in the Phillips’ curve slope coefficient $\kappa$ in the uncertainty about price stickiness and the elasticity of substitution, we will have once more that robust optimal monetary policy calls for a stronger response of the interest rate to fluctuations in inflation and the output gap.

In table 2, we show the equilibrium allocation for a different degree of parameter uncertainty and the theoretical Taylor rule coefficient $\phi_\pi$, after setting the output gap response coefficient $\phi_x$ equal to 1. The numerical exercise in the table highlights that under uncertainty about the degree of substitution between domestic and foreign goods and the degree of price stickiness, there is a very different reaction to inflation, comparing the values assumed by $\phi_\pi$. Consider the worst case of uncertainty, namely when $\eta_h = 6$ and $\theta_h = 0.95$, the analysis being tantamount for the other parameter constellations considered.

First, we can notice that a desire of robustness makes the policy-maker react much less to inflation than in the case he does not care about robustness at all. In particular, $\phi_\pi$ in the case of a robust policy is a third of the corresponding value when the followed policy is not robust. This quanti-
tative finding confirms what previously shown, namely that the Brainard principle holds in our setup. Second uncertainty implies a welfare loss, which can be offset by implementing an optimal robust monetary policy. If we focus the analysis on the parameter configuration \( \eta_h = 6 \) and \( \theta_h = 0.95 \), welfare loss amounts to 5.16 if the policy-maker is not concerned of robustness and sets interest rates as if \( P^* = P \). However, loss drops to 3.92 by taking into account uncertainty by setting interest rates to minimize the worst case outcome \( \eta_h = 6 \) and \( \theta_h = 0.95 \).

The last two rows of the table show the case of no uncertainty about the elasticity of substitution between domestic and foreign goods. Furthermore, we pick a value of \( \eta \) such that we are in the closed-economy case\(^8\). The two main conclusions of the open-economy framework carry out in closed-economy: Brainard principle holds and welfare loss is bigger if uncertainty increases. In particular, the reduction in the size of \( \phi_\pi \) is greater now than in the closed economy setup.

Summing up, the previous analysis confirms how important is not only to have a good knowledge of the true structure of the economy, but it is also important to discern how the structural parameters impact on the coefficients of the reduced form. That is why we justify a different policy reaction with respect to what found in Giannoni’s paper (2002) and other contributions.

4 Concluding remarks

This paper derives a robust optimal monetary policy rule, namely the best rule that yields a good enough performance when the true model of the economy is unknown and the policymaker chooses among different competing models. In a new Keynesian model in small open economy, uncertainty is modeled as uncertainty about the true structural parameters that characterize the economy. In particular, the central bank does not know the

\(^8\)To that extent, remember the expression of \( \omega \) and the unitary value of the relative risk aversion \( \sigma \).
true numerical values nor the statistical distribution of the degree of price stickiness $\theta$ and the elasticity of substitution between domestic and foreign goods $\eta$. He only knows the range in which the two unknown parameters are included; moreover, uncertainty about $\eta$ and $\theta$ transmits to the slope of the aggregate supply curve.

After solving for the robust optimal equilibrium allocation, we derive the robust optimal Taylor rule which allows the policy-maker to implement it. By setting this kind of Taylor rule, the policy-maker aims at minimizing the worst possible loss due to parameter misspecification. It is shown that a central bank that follows a minmax approach under uncertainty sets interest rates less aggressively to react against fluctuations in inflation or the output gap than in the case of absence of uncertainty. To that extent, the Brainard principle which prescribes cautious policy in the case of uncertainty holds.

The latter finding is a crucial result of the paper and is in contrast with the more recent contributions about the effects of model uncertainty to the optimal interest rate setting. In particular, our results are very differently from the ones sketched out in a relevant paper by Giannoni (2002). At a first sight, we have interpreted this different finding because of the open-economy
dimension of our model, which gives the policy-maker the incentive to use the terms of trade strategically. By considering the limiting case of closed economy, however, we find that the Brainard principle continues to hold and we explain that with the importance of understanding the key parameters which hinder a correct evaluation of the slope of the Phillips curve.

It remains to analyze how these results are sensitive to the way information structure is modeled. A competing way to model uncertainty assumes that the policy-maker at least knows the distribution, as in the methodology developed by Hansen and Sargent. Moreover, in order to better capture empirically the monetary policy trade-offs in open economy, we should abandon some limiting assumption in our model, namely the presence of complete exchange rate pass-through. We leave this issue for future research.

References


A Determinacy properties for Taylor rules

In order to derive the determinacy properties of the robust optimal Taylor rule, we substitute (8) into (1) and (2). Rearranging the resulting equations we obtain

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + B \begin{bmatrix} \pi^n_t \\ \varepsilon_{\pi_t} \end{bmatrix}$$

(26)

where

$$A = \begin{bmatrix} 1 + \frac{(1+\omega)^{1+\phi}}{\sigma} & \frac{(1+\omega)\phi}{\sigma} \\ -\frac{1}{\beta} & \frac{1}{\beta} \end{bmatrix}$$

and

$$B = \begin{bmatrix} \frac{1+\phi}{\sigma} & \frac{1+\omega}{\beta \sigma} \\ 0 & -\frac{1}{\beta} \end{bmatrix}.$$  

In order ensure determinacy, we need both the eigenvalues to lie outside the unit circle. The characteristic polynomial is $\lambda^2 + A_1 \lambda + A_0$, with $A_1 \equiv -tr(A)$ and $A_0 \equiv det(A)$.

Solving the characteristic polynomial and using the standard methodology\(^9\) it is possible to derive the determinacy condition (22).

\(^9\)See for example, Woodford (2003).
Figure 1: Loss as a function of the elasticity of substitution $\eta$
Figure 2: Loss as a function of the degree of price stickiness $\theta$

Figure 3: Determinacy condition for a simple Taylor rule which implements the optimal minmax equilibrium allocation